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Exchange statistics, charge detection and back-action dephasing by a mesoscopic beam collider

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Abstract
We investigate properties of charge detection and detection-induced dephasing of a charge qubit interacting with an electron beam collider composed of a quantum point contact. We predict that the interference of the qubit is fully restored when the two inputs are identically biased so that all the electrons suffer two-electron collision, unlike the case without collision. This phenomenon is related to Fermi statistics and illustrates the peculiar nonlocality of dephasing. We also describe the detection properties of entangled electron pairs. For singlet pairs, the sensitivity of charge detection is enhanced which originates from the bunching behavior of electrons. This demonstrates that control of exchange statistics of particles can help sensitive charge detection.

In a two-path interferometer with a *which-path* (WP) detector, the observation of interference and acquisition of the WP information are mutually exclusive [1–3]. It has been shown that dephasing (i.e. reduction of the interference) can be understood either as the acquisition of the WP information or as the back-action caused by the detector [3]. However, it has been argued that the back-action dephasing has not simply occurred as a result of the classical momentum kick in some cases [4–7]. Owing to the recent advances in nanotechnology, mesoscopic devices now provide opportunities for investigating this issue. Indeed, WP detection in quantum interferometers has been achieved by using mesoscopic conductors [7–11]. In these experiments, a quantum point contact (QPC) was used as a WP detector by probing the charge of a single electron at a nearby quantum dot [7–10] or a ballistic two-path conductor [11]. The particular set-up we consider here is schematically drawn in figure 1: a charge qubit interacting with a QPC beam collider having four (two input and two output) electrodes. It has been theoretically well understood that the dephasing of the qubit is caused by the charge detection when uncorrelated electrons are injected from only one of the source electrodes and partitioned by the QPC (that is, when the QPC does not suffer from electron collision) [6, 12–19].

In this paper, we report our investigations of the dephasing properties of the qubit when the detector electrons, injected from the two input electrodes, collide at the QPC. We find that the dephasing is suppressed (i.e. the interference is preserved) as a result of the two-particle collision, in spite of charge sensitivity of the scattering coefficients at the QPC. When the two electrons, injected from the two different inputs, collide at the QPC, Fermi statistics leads to antibunching of electrons.

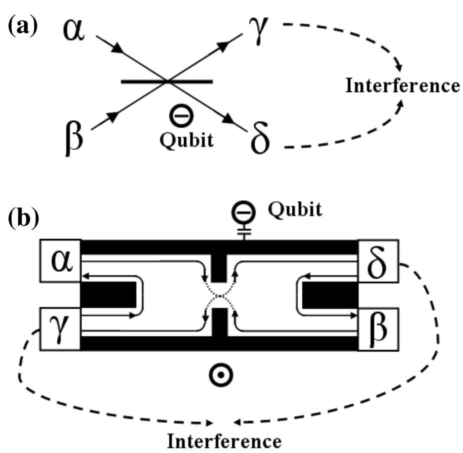

Figure 1. (a) A schematic diagram of a charge qubit electrostatically coupled to a detector composed of a beam collider and four (two input and two output) electrodes, (b), which can be realized, for instance, by using the quantum Hall bar and quantum point contact.

As a result, two electrons coming from the two input leads are transferred to the two different output leads because of Pauli's exclusion principle manifested in two-particle interference. The antibunching of electrons makes it impossible, even in principle, to extract the information of the charge state. We argue that this shows the nonlocal nature of dephasing. In the case that the two input electrodes emit entangled pairs, charge detection sensitivity depends on the parity of the two-particle state. For triplet pairs, the exchange statistics are equivalent to that of independent fermions. However, for singlet pairs, the two-particle interference leads to bunching of electrons, which enhances the charge sensitivity. Our prediction indicates two important features: (1) a particle-like behavior of a quantum state emerges from the information itself, rather than from interaction-induced disturbance [6, 7] and (2) the control of exchange statistics can be used to achieve higher charge sensitivity of a mesoscopic detector.

The system under consideration is composed of a charge qubit interacting with a QPC detector having four electrodes (figure 1(a)). This kind of detector can be constructed with the quantum Hall bar and split gates as schematically drawn in figure 1(b). One could also make use of the interference between the two output beams (dashed lines of figures 1(a) and (b)) for a phase-sensitive charge detection. Constructing interference [20] far away from the qubit does not influence dephasing of the qubit, but controls the efficiency of detection [18, 21]. The electron spin is neglected at this moment. (Charge detection with spin-entangled electrons is discussed later.) The qubit, composed of two states, namely $|0\rangle$ and $|1\rangle$, may either be a quantum dot [7, 8], double quantum dot [9] or a two-path interferometer [11]. Creation (annihilation) of an electron at each electrode x ($\in \alpha, \beta, \gamma, \delta$) is represented by the operator $c_x^\dagger$ (c_x). The characteristics of the scattering of an electron at the QPC is accounted for by the scattering matrix

$$S_i = \begin{pmatrix} r_i & t_i' \\ t_i & r_i' \end{pmatrix}, \tag{1}$$

depending on the charge state i ($\in 0, 1$) of the qubit, which transforms the electron operators as

$$\begin{pmatrix} c_\gamma \\ c_\delta \end{pmatrix} = S_i \begin{pmatrix} c_\alpha \\ c_\beta \end{pmatrix}. \tag{2}$$

Charge detection and dephasing induced by the detection have been extensively studied previously when one of the input electrodes injects uncorrelated electrons [6, 12–19]. In our set-up of figure 1, this situation can be reproduced when one of the input electrodes is biased and all the other electrodes are grounded. For later purposes, first we briefly review the detector-induced dephasing in this case. When an electron is injected from input α, the wavefunction, $|\psi\rangle$, is composed of the individual wavefunctions of the qubit, $a_0|0\rangle + a_1|1\rangle$, and the detector state, $c_\alpha^\dagger|F\rangle$. ($|F\rangle$ denotes the Fermi sea of all the electrodes with energy lower than zero.) Upon interaction of the detector electron with the qubit, it evolves as

$$(a_0|0\rangle + a_1|1\rangle)\otimes c_\alpha^\dagger|F\rangle \to a_0|0\rangle\otimes|\chi_0\rangle + a_1|1\rangle\otimes|\chi_1\rangle, \tag{3}$$

where $|\chi_i\rangle = (r_i c_\gamma^\dagger + t_i c_\delta^\dagger)|F\rangle$ ($i = 0, 1$). This results in an evolution of the reduced density matrix ρ of the qubit, $\rho = Tr_{\mathrm{det}}|\psi\rangle\langle\psi|$, obtained by tracing over the detector states of equation (3):

$$\rho_{ij} = a_i a_j^* \to a_i a_j^* \langle\chi_j|\chi_i\rangle = a_i a_j^*(r_i r_j^* + t_i t_j^*). \tag{4}$$

This leads to suppression of ρ_{ij} for $i \neq j$, which gives rise to dephasing of the qubit state upon continuous injection of detector electrons.

Now, let us consider the situation when electrons are injected from both of the input electrodes α and β so that two electrons collide at the QPC. In this case, the initial detector state, $c_\alpha^\dagger c_\beta^\dagger|F\rangle$, evolves into

$$|\chi_i\rangle = (r_i c_\gamma^\dagger + t_i c_\delta^\dagger)(t_i' c_\gamma^\dagger + r_i' c_\delta^\dagger)|F\rangle,$$

where i denotes the charge state of the qubit (being $i = 0$ or 1). Considering Fermi statistics, $\{c_x, c_y^\dagger\} = \delta_{xy}$, we find

$$|\chi_i\rangle = (r_i r_i' - t_i t_i')c_\gamma^\dagger c_\delta^\dagger|0\rangle = \mathrm{e}^{\mathrm{i}\theta_i} c_\gamma^\dagger c_\delta^\dagger|F\rangle, \tag{5}$$

where $\theta_i = \arg(r_i r_i') = \arg(t_i t_i') + \pi$ is the global phase of S_i. The latter equality of equation (5) is a result of the unitarity of S_i. As a result of two-particle interference and Fermi statistics, the detector state of equation (5) has only one particular possibility, that each electron propagates into different output leads, γ and δ, respectively (so-called 'antibunching'). This implies that the evolution of the density matrix of the qubit is given as

$$\rho_{ij} = a_i a_j^* \to a_i a_j^* \langle\chi_j|\chi_i\rangle = a_i a_j^* \mathrm{e}^{\mathrm{i}(\theta_i - \theta_j)}. \tag{6}$$

The magnitude of off-diagonal components of the density matrix is invariant upon collision. *Therefore, the two-particle collision in the detector does not reduce the interference, unlike in the case of single-particle scattering.*

To be specific, we consider a general case with many electrons injected from the two input electrodes α and β biased by V_α and V_β ($V_\alpha \geqslant V_\beta > 0$), respectively. The two output electrodes are grounded ($V_\gamma = V_\delta = 0$). The state of the composite qubit–detector system initially given as

$$(a_0|0\rangle + a_1|1\rangle)\otimes\left[\prod_{0<\varepsilon\leqslant eV_\beta} c_\alpha^\dagger(\varepsilon)c_\beta^\dagger(\varepsilon) \prod_{eV_\beta<\varepsilon'\leqslant eV_\alpha} c_\alpha^\dagger(\varepsilon')|F\rangle\right], \tag{7}$$

gets entangled upon interaction between the two subsystems as

$$a_0|0\rangle\otimes\prod_{0<\varepsilon\leqslant eV_\alpha}\chi_0^\dagger(\varepsilon)|F\rangle + a_1|0\rangle\otimes\prod_{0<\varepsilon\leqslant eV_\alpha}\chi_1^\dagger(\varepsilon)|F\rangle, \tag{8}$$

where ($i = 0, 1$)

$$\chi_i^\dagger(\varepsilon) = \begin{cases} \mathrm{e}^{\mathrm{i}\theta_i} c_\gamma^\dagger(\varepsilon)c_\delta^\dagger(\varepsilon) & 0 < \varepsilon < eV_\beta \\ r_i c_\gamma^\dagger(\varepsilon) + t_i c_\delta^\dagger(\varepsilon) & eV_\beta < \varepsilon < eV_\alpha. \end{cases} \tag{9}$$

From this we obtain the time dependence of the reduced density matrix:

$$\log[\rho_{ij}(t)] = \log[\rho_{ij}(0)] + \sum_\varepsilon \log\lambda_{ij}(\varepsilon), \tag{10}$$

where $\lambda_{ij}(\varepsilon)$ corresponds to the indistinguishability parameter of the detector electrons with energy ε (just as in $\langle\chi_j|\chi_i\rangle$ for the simpler cases in equations (4) and (6)). We find

$$\lambda_{ij}(\varepsilon) = \langle F|\chi_j(\varepsilon)\chi_i^\dagger(\varepsilon)|F\rangle$$
$$= \begin{cases} \mathrm{e}^{\mathrm{i}(\theta_i-\theta_j)} & 0 < \varepsilon < eV_\beta \\ r_j^* r_i + t_j^* t_i & eV_\beta < \varepsilon < eV_\alpha \\ 1 & \text{otherwise.} \end{cases} \qquad (11)$$

At time $t \gg h/eV_\alpha$, where the energy–time phase space is much larger than h, the summation $\sum_\varepsilon$ can be replaced by $t\int \mathrm{d}\varepsilon/h$. In this limit, we obtain $|\rho_{01}(t)| = |\rho_{01}(0)|\exp(-\Gamma_\mathrm{d}t)$ with the dephasing rate Γ_d given by

$$\Gamma_\mathrm{d} = -\int \frac{\mathrm{d}\varepsilon}{h}\log|\lambda_{01}(\varepsilon)|, \qquad (12)$$

and we get

$$\Gamma_\mathrm{d} = -\frac{e|V_\alpha - V_\beta|}{h}\log|r_0 r_1^* + t_0 t_1^*|. \qquad (13)$$

Note that the effect of inelastic scattering in the detector is not considered here. The contribution of inelastic scattering to dephasing is negligible if the electron escape rate in the qubit is much smaller than $eV/\hbar$, which is the case for a weak continuous measurement. In the weak coupling limit ($r_0 r_1^* + t_0 t_1^* \sim 1$), the dephasing rate can be expanded in terms of the change in the transmission probability, $\Delta T = |t_1|^2 - |t_0|^2$, and the change in the relative scattering phase $\Delta\phi = \arg(t_1/r_1) - \arg(t_0/r_0)$. This expansion results in

$$\Gamma_\mathrm{d} = \Gamma_T + \Gamma_\phi, \qquad (14)$$

$$\Gamma_T = \frac{e|V_\alpha - V_\beta|}{h}\frac{(\Delta T)^2}{8T(1-T)}, \qquad (15)$$

$$\Gamma_\phi = \frac{e|V_\alpha - V_\beta|}{2h}T(1-T)(\Delta\phi)^2, \qquad (16)$$

where $T = (|t_1|^2 + |t_0|^2)/2$.

Equations (13)–(16) are our central result for the collision of electrons from independent sources. When only one of the input electrodes α injects electrons, that is for $V_\alpha > 0$ and $V_\beta = 0$, equations (13)–(16) are reduced to the previously studied dephasing rate through partitioning the uncorrelated electrons [12–19]. Turning on the bias of the other input β results in the decrease of the dephasing rate in spite of the increasing number of detector electrons. For identical input biases, $V_\alpha = V_\beta$, the dephasing rate vanishes (note that the system is not in equilibrium since we are considering the limit $V_\alpha = V_\beta > V_\gamma = V_\delta = 0$). This intriguing result originates from the two-electron collisions which do not reduce the interference in the qubit and can be understood as follows. As shown in equation (5), two electrons cannot scatter into the same output lead because of Fermi statistics. This 'antibunching' makes the transport noiseless [22]. Therefore, output currents at leads γ and δ are insensitive to the charge state of the qubit (ΔT in the scattering coefficients plays no role). Furthermore, the phase sensitivity $\Delta\phi$ does not affect the detector in any noticeable way when an interferometer is constructed between the two output leads. Therefore, *charge detection is impossible, even in principle, through the two-electron collision.*

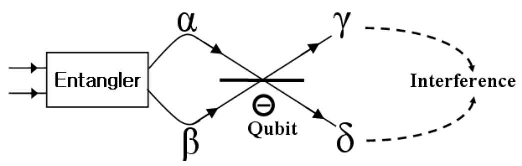

Figure 2. A schematic diagram of a charge qubit and a detector that injects spin-entangled electrons.

Our result indicates the *nonlocality of dephasing*. The origin of dephasing can be interpreted either by information acquisition in the detector, or by back-action of the detector, causing the random fluctuation of the phase in the qubit [3]. The 'back-action dephasing' is often identified with 'momentum kick' or local 'disturbance' imposed by the uncertainty principle [1]. In the 'back-action' interpretation, one might be tempted to assume a picture that the local Coulomb interaction exerts force (or a momentum kick) to the qubit, leading to uncertainty of the phase. However, our result shows that this naive picture should be discarded. Injecting additional electrons at lead β does not affect the scattering matrix of equation (1) as long as lead β is far apart from the qubit. If the local disturbance were the only origin of dephasing, increasing V_β would always monotonically raise the dephasing rate due to the increment of detector electrons. However, as we find above, the two-electron collision does not contribute to dephasing in spite of charge sensitivity of the scattering matrix, and it is a result of the nonlocality of dephasing. We emphasize that the particle-like behavior of the qubit emerges only when the charge state information could be acquired in the detector, even if it could be done only in principle [6, 7, 21].

It should also be noted that this nonlocality is not just a result of an extended wavefunction through the different leads. Let us consider a situation where the wavepacket size (which is inversely proportional to the bias voltage) is much smaller than the conducting regions between the leads. Our prediction is still valid in this limit, and thus the nonlocality does not simply mean an extended wavefunction. In other words, our result does not depend on whether or not the wavepacket extends over the different leads.

Next, let us consider injection of spin-entangled electrons from the two input leads identically biased with V (figure 2). Some possible implementations of the spin-entangled electrons in solid-state circuits are found in [23]. The 'entangler' injects spin-entangled electrons to the leads α and β. The scattering matrix at the QPC is assumed to be spin-independent and is given by equation (1). The injected entangled triplet (singlet), prior to scattering at the QPC, is written as [24]

$$\frac{1}{\sqrt{2}}(c_{\alpha\uparrow}^\dagger c_{\beta\downarrow}^\dagger \pm c_{\alpha\downarrow}^\dagger c_{\beta\uparrow}^\dagger)|F\rangle, \qquad (17)$$

where $\uparrow$ and $\downarrow$ represent the spin state of an electron. The $+(-)$ sign in equation (17) corresponds to the triplet (singlet) state. Upon collision at the QPC it is reduced to the qubit-charge-dependent detector state $|\chi_i^{t(s)}\rangle$ given by

$$|\chi_i^{t(s)}\rangle = \frac{1}{\sqrt{2}}\{(r_i c_{\gamma\uparrow}^\dagger + t_i c_{\delta\uparrow}^\dagger)(t_i' c_{\gamma\downarrow}^\dagger + r_i' c_{\delta\downarrow}^\dagger) \pm (r_i c_{\gamma\downarrow}^\dagger + t_i c_{\delta\downarrow}^\dagger)(t_i' c_{\gamma\uparrow}^\dagger + r_i' c_{\delta\uparrow}^\dagger)\}|F\rangle.$$

Again, Fermi statistics, $\{c_{i\sigma}, c_{j\sigma'}^\dagger\} = \delta_{ij}\delta_{\sigma\sigma'}$, is crucial in characterizing the detector properties. We find that the triplet state is simplified as

$$|\chi_i^t\rangle = \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\theta_i}(c_{\gamma\uparrow}^\dagger c_{\delta\downarrow}^\dagger + c_{\gamma\downarrow}^\dagger c_{\delta\uparrow}^\dagger)|F\rangle, \tag{18}$$

which leads to the indistinguishability parameter λ_{ij} of equation (10) as

$$\lambda_{ij}(\varepsilon) = \begin{cases} \mathrm{e}^{\mathrm{i}(\theta_i-\theta_j)} & 0 < \varepsilon < eV \\ 1 & \text{otherwise.} \end{cases} \tag{19}$$

As we find from equations (10) and (19), the dephasing rate vanishes when the input electrodes inject triplet pairs just as in the collision of independent electrons. This is again due to the antibunching of the orbital wavefunction of electrons which provides a noiseless beam upon collision. The orbital wavefunction of the triplet state is antisymmetric under exchange, and its statistics is equivalent to that of the independent fermions [24].

In contrast, the orbital wavefunction of the singlet is symmetric under two-particle exchange. Therefore we expect the detection property to be equivalent to that of bosons. Indeed, collision of the singlet pair at the QPC leads to the detector state of the form

$$|\chi_i^s\rangle = \sqrt{2}[r_i t_i' c_{\gamma\uparrow}^\dagger c_{\gamma\downarrow}^\dagger + t_i r_i' c_{\delta\uparrow}^\dagger c_{\delta\downarrow}^\dagger + \tfrac{1}{2}(t_i t_i' + r_i r_i')(c_{\gamma\uparrow}^\dagger c_{\delta\downarrow}^\dagger + c_{\delta\uparrow}^\dagger c_{\gamma\downarrow}^\dagger)]|F\rangle. \tag{20}$$

This singlet detector state, in contrary to those of the triplet (equation (18)) and of the two independent electrons (equation (5)), has a 'bunching' property, which enhances the shot noise [24]. The bunching is perfect for the symmetric partitioning at the QPC (that is $|t_i| = |r_i| = 1/\sqrt{2}$), where $t_i t_i' + r_i r_i' = \mathrm{e}^{\mathrm{i}\theta_i}(|r_i|^2 - |t_i|^2) = 0$. In this case, the two electrons are always found at the same lead (γ or δ). Moreover, this bunching enhances the charge sensitivity of the detector. For the detector injecting singlet pairs, we find that the dephasing rate $\Gamma_\mathrm{d}^\mathrm{s}$ is given as

$$\Gamma_\mathrm{d}^\mathrm{s} = -\frac{2eV}{h}\log|\lambda_{01}^\mathrm{s}|, \tag{21}$$

where $\lambda_{01}^s = \langle\chi_1^s|\chi_0^s\rangle = 2(r_1^* t_1'^* r_0 t_0' + t_1^* r_1'^* t_0 r_0') + (t_1^* t_1'^* + r_1^* r_1'^*)(t_0 t_0' + r_0 r_0')$ is the indistinguishability factor for a singlet pair. The factor 2 on the right-hand side of equation (21) comes from the spin degeneracy, which was not taken into account in equation (13). In the weak measurement limit, $\Gamma_\mathrm{d}^\mathrm{s}$ is given by an algebraic sum of the two different contribution: $\Gamma_\mathrm{d}^\mathrm{s} = \Gamma_T^\mathrm{s} + \Gamma_\phi^\mathrm{s}$, where the current-sensitive (Γ_T^s) and the phase-sensitive (Γ_ϕ^s) contributions are given as

$$\Gamma_T^\mathrm{s} = \frac{eV}{h}\frac{(\Delta T)^2}{T(1-T)}, \tag{22}$$

$$\Gamma_\phi^\mathrm{s} = \frac{eV}{h}4T(1-T)(\Delta\phi)^2. \tag{23}$$

The dephasing rate is now enhanced (by eight times) compared to the case with only one electrode injecting uncorrelated electrons ($V_\beta = 0$, $V_\alpha = V$ in equation (16)). Taking into account the simultaneous injection from the two inputs and the spin degeneracy, the number of injected electrons for a given time is four times larger in the case of injecting singlet states. This means that the charge sensitivity of the singlet pairs is twice that compared to the uncorrelated single electrons. It is noteworthy that this scheme can be utilized to achieve more precise charge detection in realistic devices[1].

In conclusion, we have analyzed the properties of charge detection in a QPC when the electrons from different inputs collide. We have found that the properties of dephasing are determined by the statistics of the incident electrons, and demonstrated the nonlocality of dephasing. This verifies that, while the dephasing is directly related to the which-path information in general, it cannot be simply understood in terms of a local disturbance that washes out the coherence. We have also pointed out that control of exchange statistics can be utilized to make a charge detector with higher sensitivity.

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